TP – MASTER 2 TMA

Active-grid-generated turbulence

I. INTRODUCTION

Spectral analysis is a mathematical tool present in several applications, particularly in signal processing (Figure 1). It is at the core of several techniques for describing these signals in the frequency domain. In this lab session we will study numerically and experimentally the role of the Fourier transform and the power spectral density (PSD) in different typical signals and in signals obtained with a hot wire. In particular, data acquired will help to visualize and understand several properties of turbulent flows.

Figure 1: portrait of Joseph Fourier (left), 'prefect' of Isère between 1802 and 1815. His work, 'Theory of heat analysis', appeared in 1822, is the starting point of Fourier's analysis[1].

Fourier transform

The Fourier transform of a continuous signal $x(t)$is defined as:

$$\hat{X}(\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt,$$

where we can also define the inverse transform as,

$$x(t) = F^{-1}\{\hat{X}(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{X}(\omega)e^{-j\omega t}d\omega.$$

Energy spectral density

The Fourier transform of a signal is, most of the time, a complex number. To represent it, it is therefore necessary to plot the modulus and phase of the signal $\hat{X}(\omega)$.

It can be shown that the energy of the signal, which is written by definition as the integral of the energy transported at each instant t, can also be expressed as the integral of the energy contributions transported by each pulse or frequency: this gives us the continuous version of Parseval's theorem:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \left|\hat{X}(\omega)\right|^2 d\omega = \int_{-\infty}^{\infty} \left|\hat{X}(f)\right|^2 df$$

with f the frequency in Hz. We will then call $\left|\hat{X}(f)\right|^2$ the energy spectral density of the signal.

When a signal has infinite energy, but finite power, we can also a **power spectral density (PSD):**

$$PSD = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left|\hat{X}(f)\right|^2 df$$

Note: all these definitions concern continuous-time signals, but they are easily adapted for discrete ones.

The goal of this lab session is to study how the spectral analysis can help us to study experimental signals in a lab. It will also show the advantages and limitations of such analysis.

Homogeneous Isotropic turbulence.

During this session we will assume previous basic knowledge on turbulent flows: Kolmogorov spectra, some of the scaling proposed by him and relevant length scales and non-dimensional numbers (integral, Kolmogorov and Taylor scales, turbulent Reynolds number Re_λ, 5/3 law, etc…). Further information can be found in Pope's book[6].

I.1 Some basic commands in Matlab/Python.

During this session, we will use Matlab or Python to do the spectral analysis. On the following, we detail the commands you will need to use[2]:

- The algorithm **fft** (Discrete Fast Fourier Transform): it computes the Fourier transform of a vector. The output is a complex number.

- The algorithm **pwelch**: part of the signal processing toolbox. It computes the power spectral density using the Welch method. The output is a positive real number.

- Matlab also has random number generators: a uniform generator over the range (0,1) that works with the command **rand** and a Gaussian generator (centered and with variance 1), that works with the command **randn**.

- Basic commands like: **mean**, **std**, **plot**, **loglog, linspace,** etc[3]…

Further information and help about these and other command scan be accesses with he commands **help** and **doc**.

I.2 Preliminary questions.

- How can we estimate the PSD from the fft output?

- How can we find the frequency vector with the fft? (help: the Matlab/Python doc has the answer!).

- One way to use pwelch algorithm on a given vector YY is with the options:

  [P,f]=pwelch(YY-mean(YY),hanning(Nfft),Nfft/4,Nfft,Fs) ;

  where **Nfft** is the number of points for each averaged fft segment, **Fs** is the sampling frequency and **hanning** the window function used to do the superposition. **P** is the resulting PSD **f** the frequency vector in **Hz**. Explain the role and relevance of each term (**warning**: this equation is valid for Matlab, check Python's doc for possible minor variations).

- What form will the PSD have for a signal obtained with rand? Note: this signal is known as 'white noise'.

II. THE FACILITY AND THE TESTS

II.1 LEGI's wind tunnel

The LEGI has a large subsonic wind tunnel (figures 2 and 3). It is able to generate a laminar flow in the test section ($0.75cm^2$ in section and 4m in length) with speeds between 5 m / s and 50 m / s. A small wind tunnel (which can go up to 40 m / s) is also available for this lab.
We will use the large wind tunnel with an active grid, which makes it possible to generate homogeneous and isotropic turbulence with high intensity[4]. The small wind tunnel will be used empty (laminar flow) and/or with a cylinder to study the spectral response of the hot wire.

Attention: do not start any wind tunnel before you have been briefed by the supervisors!

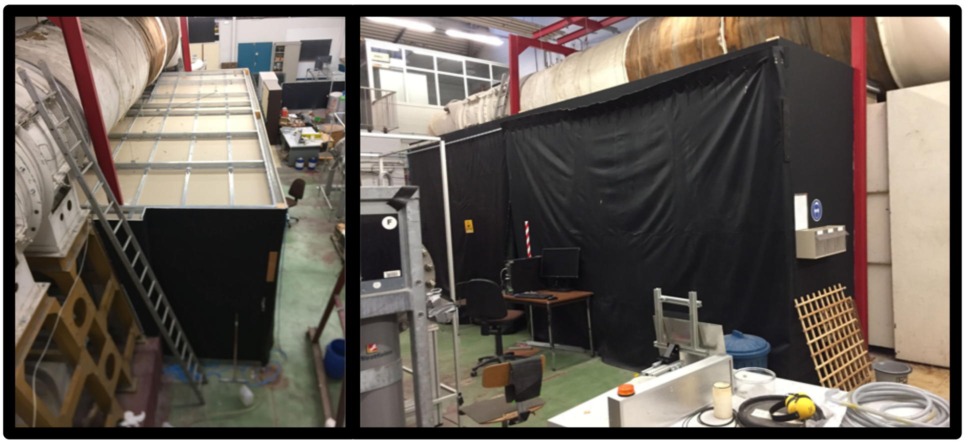

Figure 2: LEGI's large wind tunnel

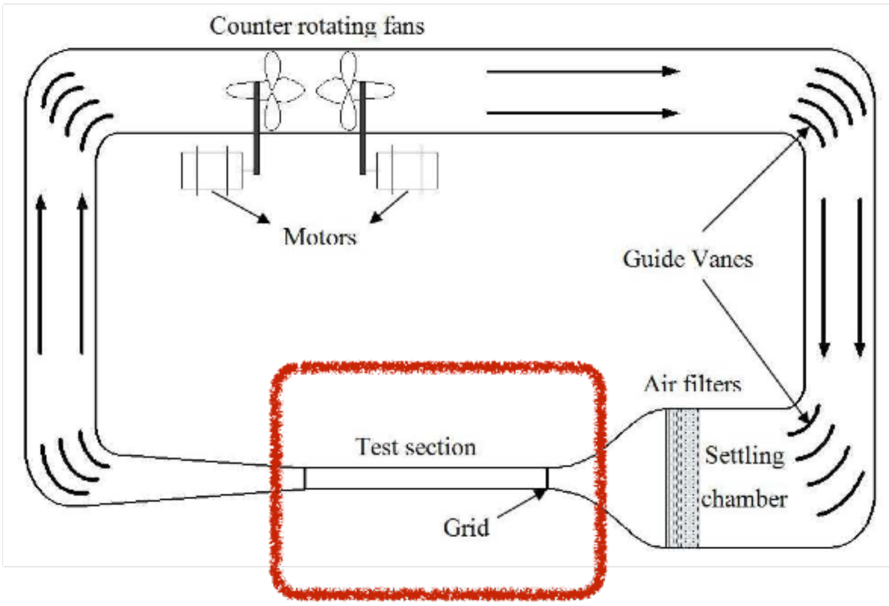

Figure 3: schema of the large LEGI wind tunnel

II.2 THE ACTIVE GRID

Active grids are widely used as they allow to generate bespoke turbulence (gusts, shear profiles, large Re turbulent flows, etc…). The active grid available at LEGI (figure 4) is composed by 16 rotating axes (eight horizontal and eight vertical) mounted with co-planar square blades and a mesh size of $M = 10$cm, (see reference 5 for further details about the active grid). Each axis is driven by a motor whose rotation rate and direction can be controlled independently. Two protocols were used with the active grid. In the active grid protocol (also referred to as ``triple-random" in the literature), the blades move with random speed and direction, both changing randomly in time, with a certain time scale provided in the protocol. For the open-grid protocol, each axis remains completely static with the grid fully open, minimizing blockage. These two protocols have been shown to create a large range of turbulent conditions, from $Re_\lambda \sim 30$ for the open/static mode to above 800 for the triple random one.

Figure 4: picture of the active grid[5].

III. MEASURES

III.1 HOT WIRE CALIBRATION

Hot-wire anemometry is an experimental Eulerian technique classically used in turbulence. It consists in placing in the flow a cylindrical bar of small size (typically a few μm) overheated compared to the environment. The measurement is based on the principle of forced convection where the flow acts as a cooler. The heat exchange law thus makes it possible to link the fluctuating velocity field of the flow with the heat loss of the wire. The wire can be operated in two modes: constant current or constant temperature. In this lab session we will always work in constant temperature anemometry (CTA, constant temperature anemometry).

All hot wires have to be calibrated before use, using King's law (see below). For that, the output of the hot wire (in volts) is compared to a velocity reference. In our case, the latter comes from a Pitot tube capable of giving the average velocity in the wind tunnel. The calibration will be done following these steps:

- We will acquire using the sampling frequency 20kHz. Which is the number of samples N needed to acquire a signal of 10s?
- Set these parameters on the acquisition program.
- Check the resistance and voltage of the HW with no incoming velocity.
- Start the wind tunnel. Which is the order of the wire's voltage?
- Note the average velocity (from the Pitot tube) and voltage (from the hot wire) for the velocities: 1,2,4,5,7.5,10 and 15 m/s (estimated approximately using the wind tunnel's motors rotation frequency).
- Propose a calibration using or a 4[th] order polynomial or King's law:

$$e^2 = A\sqrt{u} + B,$$

With e the voltage from the wire, u the velocity and A and B the calibration parameters.

State the advantages/disadvantages of each method.

III.1 EXPERIMENTAL WORK

- We will now acquire using the sampling frequency 20kHz. Which is the number of samples N needed to acquire a signal of 120s?
- Set these parameters on the acquisition program. Put the hot wire at, at least $30M$ from the grid (in order to achieve HIT conditions).
- We will study two protocols: triple random and open/static.
- We will also do at least two freestream velocities to check dependencies with the Reynolds number.
- If time allows: do you expect turbulence properties to depend on the streamwise distance?

Important: verify the spectra and averaged values obtained for each case. Save all data in '.mat' format so it can be used in the next, post-processing, session. The data that has to be saved are the time signals of voltage (or velocity) for the active-grid tests.

IV. SUBSEQUENT DATA ANALYSIS

IV.1 Numerical study

- With the help of the 'linspace' command, create a time vector and a vector which is a sinusoidal function of the time vector (we will call t the time vector and Y(t) the signal obtained).

- Calculate the PSD of the signal using the fft and pwelch (by varying the Nfft parameter). Interpret similarities/differences.

- Add noise to the $Y(t)$ signal with the rand command. Study the new spectrum with fft and pwelch. Comment.

- Repeat the previous item with Gaussian noise. Comment.

- Generate a random signal with the rand command. Study the PSD with the pwelch command. Now we add as 'noise' to the previous random signal an oscillatory function with a frequency of 50 Hz. What is the effect in the spectrum? Comment.

IV.2 Experimental study

- Check the turbulence intensity (defined as the standard deviation of velocity divided by its mean value) for all cases studied. Comment.
- Compute the spectra for all cases. Can you comment on the presence of a 5/3 power law? Compare both protocols and make a hypothesis in terms of the influence of Re_λ.
- Using Kolmogorov scaling for the PSD of velocity:

$$PSD_U = C_k \langle \varepsilon \rangle^{\frac{2}{3}} \kappa^{-\frac{5}{3}},$$

with $C_k \sim 0.52$, estimate the turbulent energy dissipation rate ε.

- Deduce the Taylor scale ($\lambda = \sqrt{\frac{15\nu u'^2}{\varepsilon}}$, with ν the kinematic viscosity of the flow and u' the standard deviation of velocity). Estimate Re_λ for all cases and comment on the hypothesis made above.
- Estimate the Kolmogorov length scale $\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$.
- **If time allows:** compute the integral length scale L for the autocorrelation function. Comment on Kolmogorov's formula $\frac{L}{\lambda} \propto Re_\lambda$.

V. REFERENCES

1. Fourier, Joseph. Theorie analytique de la chaleur, par M. Fourier. Chez Firmin Didot, père et fils, 1822.

2. Matlab documentation, disponible sur : www.fr.mathworks.com

3. Hahn, Brian, and Daniel T. Valentine. Essential MATLAB for engineers and scientists. Academic Press, 2016.

4. Mydlarski, Laurent. "A turbulent quarter century of active grids: from Makita (1991) to the present." Fluid Dynamics Research 49.6 (2017): 061401.

5. Mora, D. O., E. Muñiz Pladellorens, P. Riera Turró, Muriel Lagauzere, and Martin Obligado. "Energy cascades in active-grid-generated turbulent flows." Physical Review Fluids 4, no. 10 (2019): 104601.

6. Pope, Stephen B., and Stephen B. Pope. *Turbulent flows*. Cambridge university press, 2000.