Particle tracking in a turbulent oscillating grid flow

The aim of those practical sessions is to study the technique of Lagrangian particle tracking. After a brief introduction the students will have to setup the visualization environment (lighting, camera parameters and optics,…) and then make the actual recordings. In the second session, the videos will be post-processed to access particle positions in time and then compute their velocity and accelerations. Since obtaining velocities or accelerations from a trajectory is know to produce noisy signals, we will emphasize techniques to improve the differentiation process.

Practical session

I. Context

When studying the motion of a continuous medium, two distinct and complementary viewpoints can be adopted:

- Selecting a number of fixed points in the medium domain and measuring the variation in time of the fluid velocity at those locations
- Identifying a number of particles in the medium and following their path in the flow

The first approach naturally defines the *Eulerian velocity field,* $\mathbf{u}: \Omega \times \mathcal{R}^+ \to \mathcal{R}^3$ which maps any point in the fluid domain at any given time to its corresponding velocity. The second allow for defining a *Lagrangian Trajectory,* $\mathcal{T}_{\mathbf{P}_0}: \mathcal{R}^+ \mapsto \mathcal{R}^3$ which records the position of the particle at any time knowing it starts at the position $\mathbf{P}_0$. It is the second approach that we are going to illustrate through this pratical session.

In a turbulent flow, tracers exhibit chaotic trajectories due to the nonlinear dynamics dictated by the underlying Navier-Stokes equations. Tracking a large number of particles and using statistical tools to study their velocities and accelerations along their trajectory, or relative motion between two trajectories are valuable tools for characterizing the flow.

The development over the last few decades of fast imaging systems, large numerical storage capacities, and huge computational power has enabled the scientific community to achieve significant progress in the development of experimental techniques for particle tracking. These developments include both hardware materials such as cameras, lasers, and optics, as well as software, such as detection and tracking algorithms. These techniques are nowadays capable of reconstructing simultaneously the trajectories of approximately one hundred thousand particles in three dimensions within a small cubic-centimeter measurement volume at high spatial and temporal resolutions in high-Re turbulent flows.

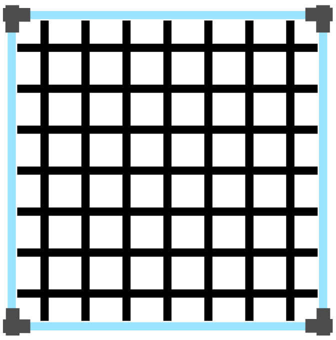
The goal of this initial 3-hour session is to introduce the experimental techniques of particle tracking through a simple oscillating grid turbulence experiment in water. We will go through the complete process, which includes selecting camera parameters, fine-tuning the optical system, seeding the flow with particles, capturing images, and choosing the tracking algorithm parameters.

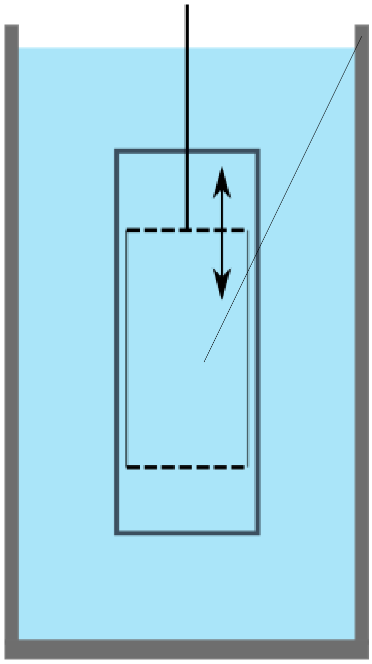
The end result will be the generation of 2D Lagrangian trajectories in a flow at various levels of Reynolds number.

II. Overview of the Experimental Setup

1- Generation of the turbulent flow

We have access to a large water tank, which can be considered as an "infinite reservoir" with well defined boundary conditions. Inside this tank, the turbulent flow will be generated by oscillating two grids within a containment tunnel. At the mid-plane between the two grids, assuming the grid velocity is sufficiently large, we expect that the generated flow will be homogeneous, isotropic, and turbulent, with no mean velocity. We use a DC gear motor together with a crankshaft to generate an approximately sinusoidal motion of the grid: the amplitude of the oscillations is fixed at $A = 18\,\mathrm{mm}$ while the frequency can be tuned by adjusting the motor voltage : 21V corresponds to (50/4.5)Hz. The grid mesh parameter (the distance between the centers of 2 adjacent bars) is $M = 15\mathrm{mm}$. The dimensions of the containment tunnel are $114 \times 114 \times 300\mathrm{mm}^3$.





In order to define a (local) Reynolds number in such flow, one needs to know the RMS velocity u_{rms} and the characteristic (integral) length scale L. With a single grid, various authors (see, e.g. Hopfinger et Al.[1]) report that the u_{rms} decreases with the distance z from the grid and is reasonably well described by a relation of the form

$$u_{rms} = C_1 \frac{\sqrt{M}(2A)^{3/2}}{z} f\ ,$$

where $C_1 \sim 0,2$. As for the integral length scale, it is known to increase linearly with z: $L \propto z$.

In the two grids case, obviously the above scaling for the integral length scale cannot hold but some recent experimental studies suggest that the linear relation between u_{rms} and the frequency remains valid.

2- Imaging System

The list of the available harware is :

- Photron High speed Camera : 1024 x 1024 pixels, 20 µm wide pixels
- Converging lens : focal length 200 mm.
- LED light
- Particles : Dantec Hollow glass Microsphere, Density = 1.06, Diameter=100um.
- A ruler.

III. Imaging the particles

Preliminary estimates:

1. The relative position of the camera and lens with regards to the center of the water tank is fixed. Take a picture of the ruler immersed in the middle of the measurement volume and make sure that it is in focus. Use that picture to
    - measure the scaling factor between image pixels and real world distances and the field of view,
    - verify that the magnification is as expected considering the geometrical arrangement of the lens and camera.
2. Preliminary measurements of the RMS velocity and the integral length scale give $u_{rms} \sim 30\,\mathrm{mm/s}$ and $L \sim 1cm$. Compute the turbulent Reynolds number and estimate the dissipative length scale η. Are the chosen particles adapted to measurements down to the smallest length scales of the flow?
3. What frame rate should we chose in order to make sure that the displacement of 99% of the particles between two frames won't exceed $\eta/5$?
4. In order to avoid obtaining blurry particles, a rule of thumb is to chose the exposure time so that the displacement does not exeeding D/10. What is the maximum admissible exposure time so that this rule is fullfilled for 99% of the particles?

Now take the time to understand how the camera software works and set the above determined parameters. Take at list one video (2 is preferable) for each of the 5 following driving voltages : [6,8,10,12,14].

IV. Particle tracking

The entire particle tracking procedure takes too much time to be handled during the practical session. You will choose the tracking parameters with the help of your teacher and the actual computations will be done afterwards, before the second session.

Data analysis session

During the second session, basic analysis of the data obtained previously will be conducted using Python. The aim is not to derive quantitative results from those data but rather to get acquainted with basic methods and typical Lagrangian velocity and acceleration statistics.

You will start with a basic jupyter notebook that loads the basic python modules and plots the trajectory of the longest track.

I. Hands on the tracks data handling

Once you have understood how track data are stored in the python record. Try to look at various tracks and see how they look.

At the end of the base notebook, we show how trajectories can be smoothed using a "gaussian blur": the technique consists in convolving the coordinates with an averaging kernel. This way coordinates are the result of an averaging with their neighbor points. The simplest averaging kernel would be [1/2 1/2], which averages the coordinates of 2 successive points, or [1/3 1/3 1/3] for 3 points ... Those rectangular kernels are known to produce artifacts and give the same weight for the central coordinate and its neighbors. For better results we use a gaussian kernel (pos_kernel) which gives more weight on the central point. Exercise : play with the width of the kernel and see its effect.

II. Derivation of the velocity and acceleration from position data

1. Compute the horizontal velocity of the the longest track with a simple first order differentiation scheme: $v_x(t) = \left(x(t+dt) - x(t)\right)/dt$ Plot the x and y velocities as a function of time. Comments.
2. Now try to do the same computation by convolving the coordinate vector with a differentiation kernel $k_{diff}(\tau)$: $v_x(t) = \int k_{diff}(\tau) x(t-\tau) d\tau$, where the kernel is simply [1, -1].

If all went well, you should have obtained the same **noisy velocity vectors** with both methods. There are a number of techniques to obtain smoother velocities, including using higher order schemes for the derivative, which take into account more points to compute the derivative, or, equivalently, larger differentiation kernels. One such kernel is often used in Lagrangian particle tracking: the derivative of a gaussian kernel (see, e.g., Mordant et Al.[2]).

3. Now compute the horizontal velocity with differentiated gaussian kernel (vel_kernel) of width 3 and support 3 times larger.

4. Do the same with a doubly differentiated gaussian kernel (acc_kernel) in order to obtain the horizontal acceleration.

III. Velocity statistics

1. Compute the mean and rms velocity of horizontal velocities (see numpy.mean and numpy.std methods) for all the forcing conditions
2. Plot those as a function of the forcing frequency.
3. Plot the Probability Density Function (see numpy.histogram method) for all forcing frequencies.
4. Plot the centered reduced PDF and compare with a normal distribution.

IV. Acceleration statistics

1. Compute the mean and rms acceleration of the horizontal component for all the forcing conditions
2. Plot those as a function of the forcing frequency.
3. Plot the reduced PDF and compare with the expected PDF if we assume that the module of the acceleration has a log-normal distribution. See Mordant et Al.[2] for details.
4. See how the acceleration kernel width affects the PDF shapes

Bibliography

[1] Hopfinger, E. & Toly, J.-A., Spatially decaying turbulence and its relation to mixing across density interfaces
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[2] Mordant, N.; Crawford, A. M. & Bodenschatz, E., Three-dimensional structure of the Lagrangian acceleration in turbulent flows
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